

Bayes & Newman (continued): a further illustration of their thought and reasoning

I was asked to give this last presentation of the calendar year at rather short notice; so I am taking advantage of the fact that the end of a year and the beginning of a new one is a traditional time for taking stock of life, by using a somewhat freer format than that of the Circle's regular monthly presentations. I am not going to present and defend a thesis; rather, I am going to pose some questions for consideration and discussion. (As I intend to travel to the Circle's meeting-pub by long-distance train on the day, this has the further advantage of giving you all something to talk about if I fail to arrive on time!)

Bayesian analysis is traditionally used for quantifying how our "reasonable degree of belief" in some proposition changes as its items of supporting evidence are accumulated. In the realm of physical measurement, with which I and several other Circle members have been concerned for most of our working lives, it is very usual to make repeated measurements of the same quantity; and mathematical analysis shows that in general an "average" (a deliberately vague term) of the multiple measurements can be given a greater degree of belief than any one of them alone. Although this is generally true, there are exceptions, both practical and philosophical. Even in the 17th-century world of Newton and Galileo, it was recognised that (for instance) the speed of a fast-moving object is a slippery concept, because it changes while you are trying to measure it. (In Einstein's view of the world, not only is it asserted that all motion is relative, but also that even the relative speed of two objects can only be measured reliably when and if they are coincident in space.) In the world of quantum mechanics, there are measurements which are simply unrepeatable because the act of measurement inevitably changes the measured system. Perhaps the most intellectually challenging of all, though, is the case of the intrinsically-rare event: if something has only happened a handful of times in recorded history, what is the chance of its happening again next week - if the recorded happenings are not just dismissed as lies or illusions?

Cardinal Newman's version of Bayesianism, "the accumulation of converging probabilities" as discussed in his "Grammar of Assent"

(https://en.m.wikipedia.org/wiki/Grammar_of_Assent), aims at demonstrating the reasonableness of belief in the ultimate "rare" event - the existence of "God" (whatever we mean by that). This is the ultimate rare event, because Newman follows Aquinas (and to some extent Aristotle and Descartes) by taking God as that entity of whom (or which) "essence" and "existence" are the same thing, an uncaused entity existing in the eternal present. Whatever you think of this conception in itself, it seems clear that there can be at most one of them; if it exists, it is responsible for the existence of anything and everything else. (CS Lewis puts this vividly by describing God as "a Being so brim-full of existence that He can give existence away, making it untrue to say that He is everything".)

I am not intending to discuss this in any detail (though Christmas seems a very appropriate time to think about it); I am using it as an example of the scope of Bayesian thinking and its ability to discuss the possibilities of rare events. I do want to open for discussion rare events whose implications are (I hope) more likely to be generally agreed-upon. Some of those I have in mind pose ethical problems as well as probabilistic ones. Here, then, are

examples which I hope are suitable for discussion even if the vagaries of public transport prevent me from being involved throughout the presentation.

1/ The supernova 1987A (https://en.m.wikipedia.org/wiki/SN_1987A)

A supernova is the explosive death of an unusually massive star (typically 10 times the mass of our Sun). Supernovae are paradoxically both very rare and quite common. For a given massive star, the supernova stage lasts a few seconds initially with a few weeks or months of "afterglow", at the end of a life measured in billions of years, so obviously the chance of seeing a supernova by choosing a favourite star and settling down to watch it is negligible. Nevertheless, so many stars are concurrently in existence (around a trillion in the "Local Group" of our galaxy and its gravitationally-associated neighbours, perhaps a trillion trillion in the whole observable universe) that the chance of seeing a supernova at some time or other is not negligible at all. The celebrated astronomer-mathematician Johannes Kepler actually lived through two; the second occurred in 1604 when he was court astronomer to the Holy Roman Emperor of the time, who required him to make progress reports on its afterglow-decay for the control of public alarm (those of us who were involved in scientific research during the recent pandemic may find this familiar!). These were later collected and published together (<https://www.digitale-sammlungen.de/en/view/bsb10873675?page=4,5>), although as they were written in imperial court Latin it is not clear how much public alarm they may have controlled (those of us who were involved in such matters as attempting to explain exponential growth during the recent pandemic from our comfort-zone of mathematics may find this familiar too).

Kepler did not understand what he was seeing; but, in the best scientific tradition (which he played no small part in creating), he left a detailed description of what he saw, hoping that it could be explained later. In 1885 another supernova appeared in our Local Group, and with the aid of spectroscopic study it was recognised as a new kind of "star". When its afterglow pattern turned out to be essentially the same as the one seen by Kepler nearly 300 years before, a new branch of astronomy began, and theories of the structure and life-cycle of stars began to grow with it - though slowly, since at that time scientists had only the vaguest ideas of sub-atomic physics and none at all of galaxies beyond our own. Other supernovae were identified, some of them retrospectively, such as the first one of Kepler's life (described in 1572 by Kepler's mentor Tycho Brahe), and the ones observed in 1006 and 1054 by the astronomers of Persia and China. All these had records of position in the sky as well as of behaviour in time, and their remnants have been found - some by accidental observation in the neighbourhood, some by deliberate search. The most recent step in the development was taken in 1987, when the burst of neutrinos emitted during the first few seconds of a supernova explosion was directly observed - just 25 of them (neutrinos are so difficult to observe that they remained hypothetical for 25 years after their existence was proposed), but enough to conform with expectations both in number and in energy.

The chief characteristic of this kind of development of physics is its flexibility in switching between small numbers of observed events and large numbers of potential causes of those rare events. Bayesian statistical methods explicitly include the concept of "inverse

probability" (https://en.wikipedia.org/wiki/Inverse_probability) which allows quantitative reasoning from observed effects to possible causes; this is in contrast to classical logic as exemplified by the reasoning of the "physico-philosophers" of ancient Greece, who mostly seemed to prefer to postulate an "ideal" cosmos and attempt to infer from it what should be observed. (Other formulations are available: e.g. "if the facts contradict my theory, so much the worse for the facts!") How do you stand on this? Why? Do you regard it as a question that can be conceivably settled, or only open to discussion for ever?

2/ The BRCA family of tumour-suppressor genes

(https://en.m.wikipedia.org/wiki/BRCA_mutation)

I discuss this problem because its scientific aspects have a very simple solution when stripped down to the mathematical essentials, allowing me to introduce a quantitative aspect to the qualitative ethical problem without (I hope) losing too many of you!

The BRCA family of genes play a very important role in preventing development of certain cancers (usually of the breast in biological women, but other diseases and genders are available). The association is so strong that women with defective genes in this family are highly likely to develop the corresponding cancer, while women with normal genes have only the usual "sporadic" risk in single-figures per cent. (If the defect is identified before a cancer has appeared, removal of much of the breast tissue has become "fashionable" as a prophylactic, as made famous by the Hollywood actress Angelina Jolie.)

Consider a family of three generations of women: grandmother, mother and teenage daughter. The grandmother develops the cancer, and during her diagnosis and treatment is found to have the genetic defect. This is known from examination of many people to be both heritable and strongly-dominant (a defective gene paired with a normal gene almost always produces a defective result). With no other evidence, therefore, the best available Bayesian estimate of the probability that the mother will also develop the cancer is $1/2$ (she may or may not inherit the defective gene from the grandmother), and the corresponding probability for the teenage daughter is $1/4$ (even if her mother has inherited the defective gene, she may or may not inherit it in turn).

Suppose that, following discussions (no doubt heated) within the family, the mother decides to refuse the offer of a genetic test both for herself and her daughter, saying she does not wish to know more than she already does. Teenage daughter, dissatisfied with this parental attitude, announces that she will have the test as soon as her mother's consent is no longer legally required. On reaching 18 (or 21, if you are following this from America), she does so - and is found to be free of the defective gene. You are a friend of the family, and the now young-adult granddaughter appeals to you for help in persuading her mother to be tested.

What advice would you give? Why?

Do you regard this as an "absolute" ethical problem, where the mother's autonomy must be respected absolutely? Or do you consider it to be a "relative" problem, where the fact that you have a measure of good news for the mother (her probability has obviously decreased) should be taken into account?

If you take the latter view, does the numerical measure of the mother's good news (her Bayesian probability has decreased from $1/2$ to $1/3$ - solution available on request) matter to you? If not, would a larger improvement matter?

Would you also take the same view if the granddaughter had been found to have the defective gene, so that it would be virtually certain that her mother also had it and therefore would develop the cancer without prophylactic treatment?

3/ The recent American presidential election

I am including this mainly in case the first two topics turn out not to use all the available time. However, some unobvious ethical aspects of it have occurred to me.

The celebrated Stephen Hawking once bet against one of his more off-the-wall ideas (with a friendly-rival physicist who didn't agree with it) on the basis that if it turned out to be wrong he would at least have the consolation prize of winning his bet. (It was right.) It was reported shortly after Trump's election success that a French bond-trader had won for his hedge fund some \$48 million by exploiting the fact that the betting odds offered on Trump in Europe (where all good liberals hoped he would fail) were significantly different from those in America (where liberals were less numerous or more hard-headed).

Do you regard this as an unethical thing to do? If so, would you have felt differently if the political colours had been reversed? Or if the trader had declared that he had personally wished for Trump to fail but wanted a consolation prize if/when he won? Or if the trader (perhaps with a smaller sum involved) had announced that he was a professional scientist who needed an insurance-policy against a victorious Trump being a disaster for American scientific research?