ON THE PHILOSOPHICAL SIGNIFICANCE OF PRIME NUMBERS

Philosophers of mathematics have been arguing for centuries whether mathematics is "discovered" or "invented". Even the language used to discuss the problem tries to face both ways at once; "invented" comes from the Latin *invenire*, which means "to find" or "to come to" (as in "to come to a conclusion"). Bertrand Russell, who was a mathematician as well as a philosopher, seemed to change his viewpoint according to his audience, saying that "mathematics, rightly viewed, possesses not only truth, but supreme beauty" but also that "mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true". (Exasperated students of the mathematics of quantum theory often find his second view congenial.)

More recently, the trend of thought has tended to be that only the purest of pure mathematical concepts are "discovered" (as already existing) while everything else is "invented" (as an intellectual game, acquiring meaning – if at all – only by elegant self-reference or grubby correspondence to some aspect of the physical world). The mathematician GH Hardy devoted an entire book (*A Mathematician's Apology*) to an exposition of this view. It is a very moving book, by a man who had always seen mathematics as a creative art and who (writing as a 60-something) was sadly aware that his youthful creative gifts were gone; but in it Hardy gives way to intellectual snobbery, implying that mathematics is degraded by being applied to the service of mankind, and that "supreme beauty" goes hand-in-hand with supreme uselessness.

Can we do anything to resolve such a conflict? It is surely best to begin at the beginning, as Descartes would do, or the mathematician and logician Leopold Kronecker – who summed up his view of the "discovered"-or-"invented" problem thus: "*Die ganzen Zahlen hat der liebe Gott gemacht, alles andere ist Menschenwerk*" ("God made the whole numbers, all else is the work of man"). The recent Nobel laureate in physics, Roger Penrose, once rebuked his long-time collaborator and friend Stephen Hawking for saying that if other universes exist we cannot possibly know anything about them, with the words "Yes we can, Stephen; in every single one of them, 13 is a prime number." I shall examine Kronecker's and Penrose's view that whole numbers in general – and prime numbers in particular – possess intrinsic properties that are relevant to any conceivable universe; and therefore must be relevant to us and our common humanity, since we live in one!

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