BAYES & NEWMAN

Introduction to a talk by Tim Hodgetts to the South London Philosophy Circle

"The actual science of logic is conversant at present only with things either certain, impossible, or entirely doubtful, none of which (fortunately) we have to reason on. Therefore, the true logic of this world is the calculus of probabilities, which takes account of the probability which is, or ought to be, in a reasonable man's mind."

This quotation is taken from the letters of the celebrated mathematical physicist James Clerk Maxwell, one of only two predecessors openly admired by the even more celebrated Einstein (the other was Newton). It neatly sums up the two systems of reasoning in the sciences: one, the classical logic described by Aristotle, in which we proceed by syllogism and reach conclusions classified as "true", "false" or "doubtful/undecidable"; the other, the "calculus of probabilities", in which we proceed by assigning probabilities or weightings to our measurements or previous provisional conclusions and combining them, so that firm conclusions are reached by the "accumulation of converging probabilities" – a phrase first used by the empiricist philosopher Locke and adopted by Cardinal Newman.

The calculus of probabilities has roots in Roman law and in the canon law of the mediaeval Catholic church, and some of the principles of those systems have survived to the present day, notably the rules of decision in criminal cases ("beyond all reasonable doubt") and in civil cases ("on the balance of probabilities"); the intermediate verdict of "not proven" in Scottish criminal cases is also a reflection of the fact that Scots law is closer than English to Roman law. However, its use as a tool of the modern mathematical sciences derives from a posthumous paper dated 1763 by the mathematician-clergyman Thomas Bayes: *An Essay towards solving a Problem in the Doctrine of Chances*; see https://en.wikipedia.org/wiki/An_Essay_towards_solving_a_Problem_in_the_Doctrine_of_Chances, which contains (in its "Outline" section) all the mathematics that is necessary to follow my presentation. Bayes' idea was rediscovered and generalised later in the 18th century by the mathematician-astronomer Laplace, who applied it to the problems of inferring orbits of planets and asteroids from small numbers of (necessarily imperfect) measurements of their positions, and it became part of the standard techniques for analysing measurements.

Conventional histories of the development of Bayesian ideas on probability often jump straight from the work of Laplace (completed in the 1820s) to the mathematician-philosophers of the 1920s, such as Frank Ramsey, John Maynard Keynes and Harold Jeffreys. The 1920s were certainly a remarkable decade for challenging established wisdoms; as well as the upheavals caused by quantum mechanics and by the serialists and atonalists in classical music, the pure mathematicians of the time set about re-examining the foundations of all branches of their subject. Rudolf Carnap, who was a logical positivist as well as a mathematician, became obsessed with treating probability using the minimum number of (rather esoteric) axioms, and disdained predecessors like Laplace who were excellent practical manipulators of algebra but less concerned with axiomatic rigour. A similar attitude inspired the French "Bourbaki" school of pure mathematics, which can be beautiful but is often sadly impractical. At the other end of the analytical scale (or possibly "meeting round the back"), philosophers of science such as Karl Popper became obsessed with the idea that no scientific theory is ever provable, asserting in effect that the "converging probabilities" can never "accumulate" or "converge" to 1 (representing certainty) but also asserting in effect that every scientific theory is falsifiable by the establishment of a single contrary fact.

In my view, both these "Popperian" ideas are false, and indeed "unscientific". In a previous talk to this philosophy Circle, I discussed the value of a single "contrary" fact (briefly, since all the mathematical sciences depend on measurement as well as theory, the degree of contrariness of a contrary fact depends on the accuracy of the measurements used to establish it; it is not simply "true" or "false"). Similarly, although "converging probabilities" can never strictly reach 1, they can become arbitrarily close to it, and the remaining difference is evaluated less by "convergence" than by our conviction that we have considered all the facts that could be significant.

Where do James Clerk Maxwell and Cardinal Newman fit into this? They both lived and worked with Bayesian ideas during the century that the conventional histories disregard, and were contemporaries of Charles Darwin, whose ideas on evolution caused as great an intellectual upheaval as anything in the 1920s. Maxwell is best known for the work that excited Einstein's admiration, uniting "electricity" and "magnetism" during the 1860s into one "electromagnetic" theory which is still in use today (and which has provided me and some other members of the Circle with many years of gainful employment). But he made his name in the 1850s, by extending Laplace's system of theoretical astronomy to show that the rings of Saturn are neither solid nor liquid but are a cloud of particles. Meanwhile, Newman was spending the 1850s and 1860s extending the ideas of Locke and Bayes into his *Essay in Aid of a Grammar of Assent* (see https://en.wikipedia.org/wiki/Grammar_of_Assent), which treats practical epistemology in a way which anticipates my objections to Popper by 150 years.

Newman's key idea in the *Grammar of Assent* is that "logic is loose at both ends". This is a non-mathematical formulation of the Bayesian principle that we need a "prior" starting-point which cannot be a proposition to be proved (otherwise we cannot escape an infinite regress), combined with an argument that we must also bridge the gap between the "accumulation of converging probabilities" and "belief beyond all reasonable doubt" by using a faculty of judgement (which Newman calls the "illative sense" – after Locke – and compares to Aristotle's concept of "phronesis"). As Newman was a Roman Catholic and (later) a Cardinal, it might be thought that any book by him would simply assume what it obviously wanted to prove; but the use of the Bayesian idea of propositions weighted by probability goes beyond such cynical or simple-minded positions. Theists and atheists will assign different prior probabilities to propositions which seem to imply that the Universe has meaning or personality, but both groups of thinkers can use Bayesian reasoning; the only people who cannot are bigots who assign to every foundational proposition a probability of 0 or 1.

I will invite you to explore these ideas with me on the day, working (if the computer hardware allows) directly from the text, which is available on-line by following the Wikipedia reference above.